Energia cinética relat:

Sub. a Forca:

$$E_c = \int_0^v F\,dl = \int_0^v \frac{dP}{dt}\,dl = \int_0^v v\,dP = \int_0^v v\,d(mv)$$

Newton:

$$E_c = \frac{mv^2}{2}$$

Relativisticamente $\quad m = m_0\gamma$

$$E_c = \int_0^v v\,d(mv) = \int_0^v v\left[v\,dm + m\,dv\right]$$

$$E_c = \underbrace{\int_0^v v^2\,dm}_{1} + \underbrace{\int_0^v m v\,dv}_{2}$$

1 - $\quad m = \dfrac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} \qquad \dfrac{dm}{dv} = m_0 (-)\frac{1}{2}\left(1-\frac{v^2}{c^2}\right)^{-3/2} \cdot \left(\frac{-2v}{c^2}\right)$

$$dm = \frac{m_0 v}{c^2}\left(1-\frac{v^2}{c^2}\right)^{-3/2} dv$$

$$\int_0^v \frac{m_0 v^3}{c^2}\left(1-\frac{v^2}{c^2}\right)^{-3/2} dv$$

// trigonométrica

$\cos(\theta) = \frac{v}{c}$ $\qquad\qquad$ $dv = -c\,sen(\theta)\,d\theta$

$$I_1 = m_0 \int_0^v \frac{v^2}{c^2}\left(1 - \frac{v^2}{c^2}\right)^{-3/2} . v\,dv$$

$$\cancel{I_1 = m_0 \int_0^v \cos(\theta)\left(1 - \cos^2(\theta)\right)^{-3/2} . c . \cos(\theta)\,d\theta \,(-)\, sen(\theta)\,d\theta}$$

$$I_1 = m_0 \int_0^v \underbrace{\cos^2(\theta)\left(1 - \cos^2(\theta)\right)^{-3/2}}\, c \cos(\theta) . (-)\, \underbrace{c\,sen(\theta)}\, d\theta$$

$$I_1 = -m_0 c^2 \int_0^v \frac{\cos^2(\theta)}{sen^2(\theta)} . \cos(\theta)\, d\theta$$

$x = sen(\theta) \rightarrow \cos^2(\theta) = 1 - x^2$

$dx = \cos(\theta)\,d\theta$

$sen^2(\theta) = x^2$

$$I_1 = -m_0 c^2 \int_0^v \frac{(1-x^2)}{x^2}\,dx = -m_0 c^2 \int_0^2 \left[x^{-2} - 1\right] dx$$

$$I_1 = -m_0 c^2 \left[-x^{-1} - x\right]_0^v = m_0 c^2 \left[\frac{1}{x} + x\right]_0^v$$

$$I_1 = m_0 c^2 \left[\frac{1 + x^2}{x}\right]_0^v = m_0 c^2 \left[\frac{1 + sen^2(\theta)}{sen(\theta)}\right]_0^v$$

$$\begin{cases} sen^2(\theta) = 1 - \cos^2(\theta) \\ \frac{v}{c} = \cos(\theta) \end{cases}$$

$$I_1 = m_0 c^2 \left[\frac{1 + 1 - \frac{v^2}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}\right]_0^v = \cancel{m_0 c^2 \left\{\frac{2 - \frac{v^2}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}} - \ldots\right\}}$$

$$\gamma = \frac{1}{\sqrt{1 - \beta^2}}$$

$$I_1 = m_0 c^2 \left[(1 + \gamma^{-2}) \cdot \gamma \right]_0^v$$

$$\boxed{I_1 = m_0 c^2 \left[\gamma + \gamma^{-1} \right]_0^v = m_0 c^2 \left[\gamma + \gamma^{-1} - 2 \right]}$$

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Outra integral.

$$I_2 = \int_0^v m v\, dv = \int_0^v \frac{m_0}{\left(1 - \frac{v^2}{c^2}\right)^{1/2}} \cdot v\, dv$$

$$\Rightarrow \frac{v}{c} = \cos(\theta) \qquad v = c\cos(\theta) \qquad dv = -c\,\mathrm{sen}(\theta)\, d\theta$$

$$I_2 = m_0 \int_0^v \frac{1}{\mathrm{sen}(\theta)} \cdot c\cos(\theta) \cdot (-) c\,\mathrm{sen}(\theta)\, d\theta$$

$$I_2 = m_0 \int_0^v (-) c^2 \cos(\theta)\, d\theta = -m_0 c^2 \int_0^v \cos(\theta)\, d\theta = -m_0 c^2 \left[\mathrm{sen}(\theta) \right]_0^v$$

$$I_2 = -m_0 c^2 \left[\sqrt{1 - \cos^2(\theta)} \right]_0^v = -m_0 c^2 \left[\sqrt{1 - \frac{v^2}{c^2}} \right]_0^v$$

$$I_2 = -m_0 c^2 \left[\sqrt{1 - v^2/c^2} - 1 \right] = m_0 c^2 \left(1 - \gamma^{-1} \right)$$

$$E_c = I_1 + I_2$$

$$E_c = m_0 c^2 \left\{ \gamma + \cancel{\gamma^{-1}} - 2 + 1 - \cancel{\gamma^{-1}} \right\} = m_0 c^2 \left\{ \gamma - 1 \right\}$$

$$E_c = m_0 c^2 \gamma - m_0 c^2 \rightarrow \boxed{m c^2 - m_0 c^2} = E_c$$

Temos então que:

$E_c = mc^2 - m_0c^2$

Cineticamente a energia total é dada por

$E_T = E_c +$ (Alguma outra envolvida)

No caso $E_T = E_c + m_0c^2$

$\rightarrow$ Como $E_c = mc^2 - m_0c^2$

$\boxed{E_T = mc^2} \longrightarrow$ mc^2 é a energia total:

~~cinética~~

$E_c = E_T -$ repouso

$\rightarrow$ m_0c^2 = energia de repouso associada à massa

$\rightarrow$ massa é energia.

Relação útil.

$P = \gamma m_0 v$

$P = \gamma m_0 v \cdot \frac{c}{c} = \gamma m_0 c \beta$

$P^2 = \gamma^2 m_0^2 c^2 \beta^2$; $E^2 = \gamma^2 m_0^2 c^4$

$m_0^2 c^4 \cdot 1 = m_0^2 c^4 \cdot \left(\gamma^2 - \gamma^2 \beta^2\right) = (m_0 c^2)^2$

obs: $\gamma^2 - \gamma^2 \beta^2 = \frac{1}{1-\beta^2} - \frac{\beta^2}{1-\beta^2} = \frac{1-\beta^2}{1-\beta^2} = 1$

$\underbrace{m_0^2 c^4 \gamma^2}_{E^2} - \underbrace{m_0^2 c^4 \gamma^2 \beta^2}_{p^2 c^2} = (m_0 c^2)^2$

$$\boxed{E^2 = p^2 c^2 + E_0^2}$$